The solutions of x6 = x3 using *Phantom Graphs*

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Most people just put x6 – x3 = 0

 x3(x3 – 1) = 0

 x = 0 or 1

Some of the better people carried on and put if x3 = 1 let x = r cis(θ)

So that r3cis(3θ) = 1cis(360n) by using De Moivre’s Theorem

This means r = 1 and 3θ = 360n so θ = 120n = 0°, 120°, 240°

(actually most people use radians but I work better in degrees)

The solutions are: $x\_{1}=1 cis\left(0\right) = $1

$x\_{2}=1 cis\left(120\right)=\frac{-1}{2}+\frac{i\sqrt{3}}{2}$

$x\_{3}=1 cis\left(240\right)=\frac{-1}{2}-\frac{i\sqrt{3}}{2}$

**(– 0.5+0.866i, 1)**

**So my special contribution was to draw the graphs y = x6 and y = x3 with their *phantoms* which shows all the solutions clearly.**



**(– 0.5– 0.866i, 1)**

**(1 + 0i, 1)**

**Incidentally, because of the Fundamental Theorem of Algebra there are 3 more solutions at the origin. The graphs actually cross 3 more times there.**